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PROJECT SPACE TRACK

A METHOD OF PREDICTING SATELLITE DECAY

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AIR FORCE SYSTEMS COMMAND
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A METHOD OF PREDICTING SATELLITE DECAY

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ABSTRACT

Equations are derived for predicting the decay day and decay revolution of a satellite. Only two parameters, obtained from the quadratic equation that predicts equatorial crossings, are used. Examples of predictions are given.

CONTENTS

1. INTRODUCTION

Equations are derived for predicting the decay day and decay revolution of a satellite. Only two parameters, obtained from a quadratic equation which predicts equatorial crossings, are used. It appears that a predicted decay day might be useful in predicting equatorial crossings. Some examples of predictions are given.

2. DERIVATION OF EQUATIONS

Let

$$\dot{P}_N = F\,(T_N) \qquad (1)$$

where $F\,(T_N)$ is a continuous function of time and $\dot{P}_N$ is the time derivative of the nodal period at revolution number N. Integration of (1)[1], by the first law of the mean for integrals, yields

$$E_{N_i} = T_{N_d} - T_{N_i} = \frac{P_{N_d} - P_{N_i}}{F(T_{N_j})} = \frac{P_{N_i} - P_{N_d}}{-\dot{P}_{N_j}}, \qquad (2)$$

where

E_{N_i} = exact number of days to decay from revolution number N_i,

N_i = revolution number,

N_d = revolution number at decay,

T_{N_d} = time at decay (in days),

[1] Figures in parentheses refer to equations.

T_{N_i} = time at revolution N_i (in days),

P_{N_i} = nodal period at T_{N_i} (in days),

P_{N_d} = period at T_{N_d} (in days),

$\dot{P}_{N_j}$ = mean value of $\dot{P}_N$ during the interval E_{N_i}, and

$T_{N_d} \geq T_{N_j} \geq T_{N_i}$.

Set

$$\dot{P}_{N_j} \equiv \frac{\dot{P}_{N_i} (P_{N_i} - P_{N_d})}{0.6\ (1 + a_{N_i})\ (P_{N_i} - \frac{88}{1440}} \tag{3}$$

where $\dot{P}_{N_i}$ is the rate of change of the period at T_{N_i}, and

$$a_{N_i} = a_{N_i}\ (P_{N_i},\ \dot{P}_{N_i},\ P_{N_d}). \tag{4}$$

The constants in (3) tend to minimize the α_{N_i} when P_{N_i} is greater than $\frac{88}{1440}$ days.

Substitution of (3) into (2) gives

$$E_{N_i} = \frac{0.6\ (1 + a_{N_i})\ (P_{N_i} - \frac{88}{1440}\ )}{-\ \dot{P}_{N_i}} \ . \tag{5}$$

For predicting the time of equatorial crossing, The National Space Surveillance Control Center (NSSCC), (L. G. Hanscom Field, Bedford, Massachusetts) uses the equation

$$T_{N + N_i} = T_{N_i} + N\,P_{N_i} + N^2\ C_{N_i} + N^3\ \ D_{N_i}\ , \tag{6}$$

where D_{N_i} is generally omitted until the last hundred or so revolutions of a satellite.

Define C_{N_i} in (6) by

$$C_{N_i} = \frac{P_{N_i} \dot{P}_{N_i}}{2} . \tag{7}$$

Define A_{N_i} by either

$$A_{N_i} = \frac{0.6 \left(P_{N_i} - \frac{88}{1440}\right)}{-\dot{P}_{N_i}} , \tag{8}$$

or

$$A_{N_i} = \frac{0.3 P_{N_i} \left(P_{N_i} - \frac{88}{1440}\right)}{- C_{N_i}} . \tag{9}$$

Substitution of (8) into (5) gives

$$E_{N_i} = (1 + a_{N_i}) A_{N_i} . \tag{10}$$

No assumed value of decay is contained in (10).

In attempting to obtain an empirical representation for a_{N_i}, through an analysis of data from 15 decayed satellites, the predictions indicated that $|a_{N_i}| << 1$ if the predictions were made when $P_{N_i} > \frac{88}{1440}$ days. When this condition is fulfilled, the accuracy of the predictions given by (9) indicates that (9) is a good approximation to the exact solution (2) for those 15 satellites.

Assume for a given satellite the P_{N_i} and C_{N_i} in (11) are such that predicted equatorial crossings are in good agreement with observations.

$$T_{N+N_i} = T_{N_i} + N\ P_{N_i} + N^2\ C_{N_i}\ . \qquad (11)$$

Substitution of these values of P_{N_i} and C_{N_i} into (9) gives the days to decay from T_{N_i}. The decay day T_{N_d} is then

$$T_{N_d} = T_{N_i} + A_{N_i}\ . \qquad (12)$$

The T_{N_d} in (12) can be used to determine C_{N_k} at some later epoch T_{N_k}. From the NSSCC bulletins we obtain T_{N_k} and P_{N_k}. Since T_{N_d} is now known, we have

$$A_{N_k} = T_{N_d} - T_{N_k}\ , \qquad (13)$$

and

$$C_{N_k} = \frac{0.3\ P_{N_k}\ (P_{N_k} - \frac{88}{1440})}{T_{N_k} - T_{N_d}}\ . \qquad (14)$$

Appendix II gives an application of this technique for determining the C_{N_k} to be used on later bulletins.

3. THE DECAY REVOLUTION

In order to predict the approximate decay revolution number N_d, use (12) to obtain T_{N_d}. Then let N_q be the revolutions from N_i to N_d and solve (15) for N_q.

$$T_{N_d} = T_{N_i + N_q} = T_{N_i} + N_q\ P_{N_i} + N_q^2\ C_{N_i}\ . \qquad (15)$$

$$N_d = N_q + N_i \; . \tag{16}$$

For a better approximation of N_d, especially when predicting far in advance of decay, the above method should be modified in the following manner. We first derive an expression for D_{N_i} in terms of P_{N_i} and C_{N_i}. This derivation requires that the P_{N_d} in (17) be assigned the value $\frac{88}{1440}$ days. When this is done, the approximate solution (9) is exactly the same as that obtained if (1) is replaced by

$$\dot{P}_N = J \left(P_N - P_{N_d}\right)^{-2/3} , \tag{17}$$

where J is a constant. From (17) we obtain

$$\ddot{P}_{N_i} = -\frac{2}{3} \left(P_{N_i} - \frac{88}{1440}\right)^{-1} \left(\dot{P}_{N_i}\right)^2 . \tag{18}$$

Define D_{N_i} in (6) by

$$D_{N_i} = \frac{P_{N_i}^{2} \; \ddot{P}_{N_i}}{6} . \tag{19}$$

Substitution of (7) and (18) into (19) gives

$$D_{N_i} = \frac{-4}{9} \; C_{N_i}^{2} \left(P_{N_i} - \frac{88}{1440}\right)^{-1} . \tag{20}$$

We substitute D_{N_i} into (21) and solve for N_q.

$$T_{N_d} = T_{N_q + N_i} = T_{N_i} + N_q P_{N_i} + N_q^{2} \; C_{N_i}$$
$$+ N_q^{3} \; D_{N_i} . \tag{21}$$

This N_q when substituted into (16) will in general give a better approximation of N_d than will the N_q obtained from (15). To illustrate the last statement we use 1959 Epsilon-2 as an example. The first two NSSCC bulletins for this satellite gives for epoch revolution number 2,

$$
\begin{aligned}
T_2 &= 46.08599 \\
P_2 &= 7.241102 \times 10^{-2} \qquad (22) \\
C_2 &= -6.795 \times 10^{-7}.
\end{aligned}
$$

Substitution of (22) into (9) gives

$$A_2 = 361.25278 \text{ days.}$$

Equation (15) becomes

$$361.25278 = N_q P_2 + N_q^2 C_2 \qquad (23)$$

From (23) we obtain $N_q = 5247$, and from (22) and (20) we obtain

$$D_2 = -1.816 \times 10^{-11}.$$

Substitution of D_2 into (21) gives $N_q = 5289$. Hence the first method gives $N_d = 5249$ and the second method gives $N_d = 5291$. Since the decay revolution was approximately 5313.5, it is quite apparent that the introduction of D_2 gives a better prediction of N_d. It should be pointed out that (20) may also serve a useful purpose in determining the values of D_{N_i} to use in (6) when predicting equatorial crossings during the last hundred or so revolutions of a satellite.

4. REMARKS

Some of the variations in the predicted decay day, (obtained by using T_{N_i}, P_{N_i}, and C_{N_i} from NSSCC bulletins), are due to the fact that bulletins must be issued even though insufficient or no observations are available on which to determine good values of T_{N_i}, P_{N_i}, and C_{N_i}. Since the predicted decay day depends only on these values, the best available values should be used. Such values are obtained by fitting (11) to the observations. An example of this is the prediction of decay for 1958 Epsilon in Appendix I.

APPENDIX I - EXAMPLES OF DECAY PREDICTIONS

1958 EPSILON (EXPLORER IV)

Equatorial crossing times for the first 937 revolutions of this satellite are contained in "Interim Definitive Orbit for the Satellite 1958 Epsilon IV " NASA TN D-410. From this reference we obtain for epoch revolution 100, T_{100} = 3.267095 August 1958. The equation

$$T_{N+100} = T_{100} + N\,P_{100} + N^2\,C_{100}$$

predicts T_{200} with zero error and T_{300} with an error of 14 seconds, when $P_{100} = 7.63746 \times 10^{-2}$ and $C_{100} = -7.84 \times 10^{-7}$. Substitution of these values into (9) gives $A_{100} = 446.1$ (days to decay from T_{100}). The decay day was approximately 23.2 October 1959, so the error in A_{100} is less than 0.4 days.

The following predictions were all obtained by using the T_{N_i}, P_{N_i}, and C_{N_i} given on NSSCC bulletins issued at epoch revolution number N_i.

1958 DELTA (Sputnik III)

N_i	A_{N_i}	T_{N_d}
7400	171.68	463.17
7600	147.66	452.52
8100	131.07	469.12
8250	115.09	463.03

8700	84.15	461.54
8900	71.88	462.24
9000	66.19	463.01
9200	53.57	463.26

The average value of T_{N_d} is 462.2. The actual decay of the satellite occurred approximately at 462.3.

1959 ZETA (Discoverer VI)

N_i	A_{N_i}	T_{N_d}
30	51.81	285.57
80	59.11	296.18
130	57.05	297.41
190	55.31	299.61
340	46.65	300.75
500	27.54	292.00
610	17.54	289.06
700	13.84	291.09
720	14.01	291.39

The average value of T_{N_d} is 293.67. The last NSSCC bulletin estimated the decay occurred about 293.75.

1959 KAPPA (Discoverer VII)

N_i	A_{N_i}	T_{N_d}
15	22.73	335.54
30	11.79	325.59
50	15.59	330.69
110	12.19	331.84
200	5.84	330.61

The average value of T_{N_d} is 330.85. The last NSSCC bulletin estimated that decay occurred shortly after 330.78.

1959 EPSILON-2 (Capsule of Discoverer V)

In Appendix II we find that the first two bulletins predict $A_2 = 361.25$ and $T_{N_d} = 407.34$. Since these bulletins gave accurate predictions on equatorial crossings, assume that the decay day is (407 ± 30). The average value of T_{N_d} obtained from the first 12 bulletins, which predict within the above limits, is 408.33. All twelve predictions were made over 225 days before decay. The actual decay was $T_{N_d} = 408.77$.

APPENDIX II - EXAMPLE OF PREDICTED DECAY DAY USED TO PREDICT EQUATORIAL CROSSINGS

In this appendix we illustrate how the predicted decay day can be used to obtain the C_{N_i} terms for later bulletins. We use 1959 Epsilon-2 (Capsule of Discoverer V) as an example. The first revolution of this satellite was arbitrarily chosen as the first revolution on 15 February 1960. The first and second bulletins issued by NSSCC gave good predictions over an interval of 173 revolutions. Both bulletins used $T_2 = 46.08599$, $P_2 = 7.241102 \times 10^{-2}$, and $C_2 =$ $- 6.795 \times 110^{-7}$. From this data we compute $A_2 = 361.25298$ and $T_{N_d} = 407.33897$. From bulletin #3, we obtain $T_{140} = 56.06614$ and $P_{140} = 7.22269 \times 10^{-2}$. Then

$$C_{140} = \frac{0.3\, P_{140}\,(P_{140} - \frac{88}{1440})}{T_{140} - T_{N_d}}$$

$$= - 6.86 \times 10^{-7} .$$

We use (11) which becomes

$$T_{200} = T_{140} + 60\, P_{140} + 3600\, C_{140},$$

to predict T_{200}. On bulletin #5, $T_{200} = 71.19978$. The error in the prediction is 21 seconds.

On bulletin #33, $T_{3000} = 256.91272$ and $P_{3000} = 6.8334 \times 10^{-2}$.

We use the above T_{N_d} and repeat the procedure to predict the epoch on bulletin #35, $T_{3100} = 263.73626$. The error in the prediction is 2 seconds.

On bulletin #48, $T_{4100} = 330.76048$ and $P_{4100} = 6.65805 \times 10^{-2}$.

We repeat the above procedure to predict the epoch $T_{4200} = 337.32994$ on bulletin #50. The error in the prediction is 24 seconds.

An application of the above to four satellites still in orbit indicates that this technique may have some value.

APPENDIX III - DECAY PREDICTION PROGRAM WITH EXAMPLES

In this appendix we discuss a program for obtaining three predictions of T_{N_d} from one epoch T_{N_i}. B_{N_i} is the T_{N_d} predicted by the T_{N_i}, P_{N_i}, C_{N_i} given on the bulletin. $\overline{B}_{N_i}$ is the T_{N_d} predicted if C_{N_i} is changed to $\overline{C}_{N_i}$ in order that (11) will predict $T_{N_{i+1}}$ with zero error. $\overline{\overline{B}}_{N_i}$ is the T_{N_d} predicted if C_{N_i} is changed to $\overline{\overline{C}}_{N_i}$ in order that (11) will predict $T_{N_{i+2}}$ with zero error.

Let $P_{N_{i+j}}$, $T_{N_{i+j}}$, $C_{N_{i+j}}$, $(j = 0,\ 1,\ 2)$, be the elements on three successive bulletins.

Compute

$$A_{N_i} = \frac{.3\, P_{N_i} \left(P_{N_i} - \frac{88}{1440}\right)}{- C_{N_i}} = - G_{N_i} \left(C_{N_i}\right)^{-1} ,$$

$$B_{N_i} = T_{N_i} + A_{N_i} ,$$

$$\overline{C}_{N_i} = \frac{T_{N_{i+1}} - T_{N_i} - (N_{i+1} - N_i)\, P_{N_i}}{(N_{i+1} - N_i)^2} ,$$

$$\overline{A}_{N_i} = - G_{N_i} \left(\overline{C}_{N_i}\right)^{-1} ,$$

$$\overline{B}_{N_i} = T_{N_i} + \overline{A}_{N_i} ,$$

$$\bar{\bar{C}}_{N_i} = \frac{T_{N_{i+2}} - T_{N_i} - (N_{i+2} - N_i)\, P_{N_i}}{(N_{i+2} - N_i)^2} ,$$

$$\bar{\bar{A}}_{N_i} = - G_{N_i} (\bar{\bar{C}}_{N_i})^{-1} , \text{ and}$$

$$\bar{\bar{B}}_{N_i} = T_{N_i} + \bar{\bar{A}}_{N_i} .$$

The equations were programmed for the IBM 709 computer. The printout gives the above eight quantities and also the orbital elements contained on the bulletins. The program starts with $i = 1$, where N_1 is the epoch revolution on the first bulletin, and then repeats the procedure for all values of i.

It was believed that the program might provide $\bar{B}$ and $\bar{\bar{B}}$ values which gave a more accurate predicted decay day than did the B values. Additional calculations are necessary before the program can be evaluated. Some predictions calculated by this program are given below.

<u>1959 LAMBDA</u> (Discoverer VIII)

$A_{780} = 39.8$	$B_{780} = 54.3$
$\bar{A}_{780} = 53.8$	$\bar{B}_{780} = 68.3$
$\bar{\bar{A}}_{780} = 51.7$	$\bar{\bar{B}}_{780} = 66.2$

Decay day was approximately 68.0.

<u>1959 EPSILON-1</u> (Discoverer V)

$A_9 = 29.1$ $\quad$ $B_9 = 255.4$

$\bar{A}_9 = 45.5$ $\quad$ $\bar{B}_9 = 271.8$

$\bar{\bar{A}}_9 = 44.7$ $\quad$ $\bar{\bar{B}}_9 = 271.1$

Decay day was between 271 and 272.

<u>1958 EPSILON</u> (Explorer IV)

$A_{4170} = 150.0$ $\quad$ $B_{4170} = 297.4$

$\bar{A}_{4170} = 141.1$ $\quad$ $\bar{B}_{4170} = 288.5$

$\bar{\bar{A}}_{4170} = 150.6$ $\quad$ $\bar{\bar{B}}_{4170} = 298.0$

Decay day was approximately 296.2.